ANSWERS

Exercise 1A PAGE 8

1 $2^3 = 8$ **2** $7^2 = 49$ **3** $49^{0.5} = 7$ **4** $10^3 = 1000$ **5** $5^4 = 625$ **6** $4^{2.5} = 32$

7 $5^{-2} = 0.04$ **8** $3^{-2} = \frac{1}{9}$ **9** $a^y = x$ **10** $b^c = y$ **11** $x^p = a$ **12** $a^3 = x$

13 $3^y = 5$ **14** $2^x = 3$ **15** $x^4 = 5$ **16** $3^p = 5$ **17** $\log_2 64 = 6$ **18** $\log_3 81 = 4$

19 $\log_9 81 = 2$ **20** $\log_9 27 = \frac{3}{2}$ **21** $\log_2 0.5 = -1$ **22** $\log_2 0.25 = -2$ **23** $\log 100 = 2$ **24** $\log 0.01 = -2$

25 $\log_p r = q$ **26** $\log_r q = p$ **27** $\log_2 y = x$ **28** $\log_3 z = y$ **29** $\log_5 4 = k$ **30** $\log_7 3 = y$

31 $\log_3 7 = p$ **32** $\log_e x = y$ **33** 2 **34** 7 **35** 4 **36** 5

37 –1 **38** –4 **39** –3 **40** –3 **41** $\frac{5}{2}$ **42** –3

43 1 **44** 0 **45** 0 **46** $\frac{5}{2}$ **47** 1 **48** 3

49 0.699 **50** 1.398 **51** 0.845 **52** 1.690 **53** 1.301 **54** 1

55 1.322 **56** 1

57 **a** Yes **b** No (If we are restricting our attention to real numbers, as we are in this unit.)

Exercise 1B PAGE 11

1 $\log(xz)$ **2** $\log(x^2y)$ **3** $\log(x^2y^3)$ **4** $\log\left(\frac{x^2}{y}\right)$ **5** $\log\left(\frac{ab}{c}\right)$ **6** $\log\left(\frac{a^3b^4}{c^2}\right)$

7 $\log(c^2a)$ **8** $\log(100x)$ **9** $\log\left(\frac{1000}{xy}\right)$ **10** $\log\left(\frac{1000y}{x}\right)$ **11** 3 **12** 4

13 3 **14** 1 **15** 2 **16** –4 **17** –1 **18** 2

19 1.5 **20** 4

21 **a** $p + q$ **b** $p + 2q$ **c** $2p + q$ **d** $p - q$ **e** $2q + 4$ **f** $p - 2q$

22 **a** $2a$ **b** $a + 2b$ **c** $a - 2b$ **d** $b + 2$ **e** $2a + b + 1$ **f** $a + 2b + 2$

23 $y = a^x$ **24** $y = 2x$ **25** $y = x^3$ **26** $y = x^{\frac{3}{2}}$ **27** $y = ax$ **28** $y = a^2x$

29 $y = \frac{1}{x}$ **30** $y = \frac{a^2}{x}$

31 **a** 75 **b** ~58 **c** ~51 **d** 9

32 **a** 3 **b** $10^{5.4}I_0$ **c** 10 **d** $10^{1.8} (\approx 63)$

33 **a** 4 **b** 4.5 (approximately) **c** 6.6 (approximately) **d** 7.8 (approximately)

e 7.4 (approximately), 0.000 005 6 moles/litre (approximately).

34 **a** 10^4I_0 **b** 10^7I_0 **c** 10^7

Exercise 1C PAGE 14

1 $x = \frac{\log 7}{\log 3}$ **2** $x = \frac{3}{\log 7}$ **3** $x = \log 27$ (i.e. $3\log 3$)

4 $x = \frac{\log 11}{\log 2}$ **5** $x = \frac{\log 17}{\log 3}$ **6** $x = \frac{\log 80}{\log 7}$ $\left(\text{i.e.} \frac{1+3\log 2}{\log 7}\right)$

7 $x = \frac{\log 21}{\log 5}$ **8** $x = \log 15$ **9** $x = \frac{\log 70}{\log 2}$ $\left(\text{i.e.} \frac{1+\log 7}{\log 2}\right)$

10 $x = \frac{\log 17}{\log 6} - 2$ $\left(\text{i.e.} \frac{\log\left(\frac{17}{36}\right)}{\log 6}\right)$ **11** $x = \frac{\log 17}{\log 3}$ **12** $x = \frac{\log 7}{\log 8} + 1$ $\left(\text{i.e.} \frac{\log 56}{3\log 2}\right)$

13 $x = \frac{\log 5}{\log\left(\frac{5}{9}\right)}$ **14** $x = \frac{\log 2}{\log 1.5}$ **15** $x = \frac{2\log 5}{\log\left(\frac{64}{5}\right)}$

16 $x = \frac{\log 6}{\log\left(\frac{8}{9}\right)}$ **17** $x = -\frac{\log 3}{\log 2}$ **18** $x = \frac{\log 3}{\log 5}$

19 $x = \frac{\log 3}{\log 2}$ **20** $x = \frac{\log 3}{\log 2}$, $x = \frac{\log 5}{\log 2}$ **21** $x = \frac{\log 7}{\log 2}$

22 **a** $\log_3 5 = \frac{\log 5}{\log 3}$ **b** $\log_2 12 = \frac{\log 12}{\log 2}$ **c** $\log_9 15 = \frac{\log 15}{\log 9}$

d $\log_9 4 = \frac{\log 4}{\log 9}$ $\left(\text{i.e.} \frac{\log 2}{\log 3}\right)$ **e** $\log_{2.5} 6.8 = \frac{\log 6.8}{\log 2.5}$ **f** $\log_{5.4} 9 = \frac{\log 9}{\log 5.4}$

23 The metal should be passed through the rollers 20 times.

24 **a** ~270 **b** ~330 **c** The population first exceeded 1000 on the 17th day ($t \approx 16.2$).

25 The risk of an accident is 51% for $a = 0.191$

26 **a** Approximately 211 000. **b** Approximately 182 000.
Sales fall to 135 000 bars per week approximately 19 weeks after the first campaign ceases.

27 **a** \$12 597.12 **b** \$17 138.24 **c** 21 years **d** **i** 17 years **ii** 14 years
e 14.9%

Exercise 1D PAGE 18

1 1 **2** -1 **3** 3 **4** 0.5 **5** $\frac{1}{3}$

6 $-\frac{1}{2}$ **7** -3 **8** $-\frac{1}{3}$ **9** $\ln 7 - 1$ **10** $\ln 50 - 3$

11 $2\ln 10 + 3$ **12** $\frac{\ln 15 - 1}{2}$ **13** $\frac{\log_e 600 + 1}{3}$ **14** $3\ln 10 - 2$ **15** $\ln 10, \ln 20$

16 $\frac{\ln 2}{\ln 7}$ **17** $\frac{\ln 3 + \ln 7}{\ln 2}$ **18** $\frac{3\ln 2 + 2\ln 5}{\ln 3}$ **19** $\frac{\ln 2 + 2\ln 5}{\ln 5}$ **20** $\frac{2\ln 3}{\ln 2 + \ln 3}$

21 $\frac{\ln 2 + \ln 3}{2\ln 3}$ **22** $\frac{\ln 3 + 2\ln 2 + 2\ln 5}{2\ln 2}$ **23** $\frac{\ln 11 + 2\ln 2 + \ln 5}{3\ln 2}$

24 $\ln\left(\frac{2000}{A}\right)$ **a** 0.288 **b** 1.386 **c** 3.689

25 **a** 2028 **b** 2045 **26** **a** 5 days **b** 9 days

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Exercise 1E PAGE 20

1 **a** $(-7, 0)$ **b** $(0, 3)$

2 $(1, 0)$ **3** $(a, 1)$

4 **a** $x = 0$, i.e. the y-axis. **b** $x = 3$ **c** $x = 0$

5 Difficult to be accurate from the graph but answers for **a** to **d** should be close to the following:

a $x \approx 2.2$ **b** $x \approx 11.2$ **c** $x \approx 3.6$ **d** $x \approx 9.1$

e Rounded to 3 dp: 2.236, 11.180, 3.624, 9.103

6 $a = 2, b = 4, c = 3$

Exercise 1F PAGE 23

1 **a** 7.19 (correct to 2 dp) **b** 1.58×10^{-10} (correct to 3 sig figs)

2 **a** Approximately 1.32 octaves **b** Higher frequency $= 8f_1$

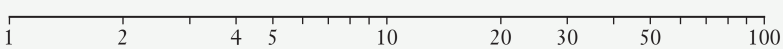
3 **a** 10^{-7} moles per litre **b** 2

4 **a** -1.39 **b** 0.98 **c** The event is more likely not to occur than to occur.

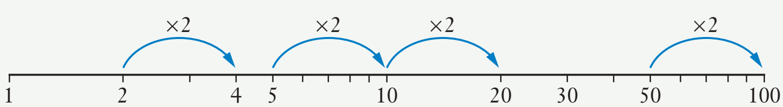
d Compare your answer to that of others.

5 Whilst it is true that the amplitude of the vibrations caused by an earthquake of magnitude 7 will be ten times that of an earthquake of magnitude 6 the cost of the damage will depend on other things too. For example, an earthquake measuring 6 on the Richter scale with its epicentre at a location of high density housing and infrastructure could cause more costly damage than an earthquake of scale 7 occurring in an uninhabited desert area. Hence the cost of damage caused by an earthquake of Richter scale 7 will not necessarily be ten times that of one with a Richter scale of 6.

6 In a logarithmic scale, if 1 to 10 is 1 unit of length then 1 to 2 will be log 2 units, 1 to 5 will be log 5 units, 1 to 20 will be log 20 units, etc. Thus on our diagram, with the distance from 1 to 10 being 5 cm the distance from 1 to 2 will be 5 cm × log 2, the distance from 1 to 5 will be 5 cm × log 5 etc.



Note also that the distance from 2 to 4 (a doubling) is the same as the distance from 5 to 10 (also a doubling) and the same as the distance from 10 to 20 and the same as the distance from 50 to 100.



This is as we would expect for a logarithmic scale because

$$\begin{aligned} \log 4 - \log 2 &= \log 10 - \log 5 \\ &= \log 20 - \log 10 \\ &= \log 100 - \log 50 \end{aligned}$$

because all are of the form

$$\begin{aligned} \log(2a) - \log a &= \log(2a \div a) \\ &= \log 2 \end{aligned}$$

Miscellaneous exercise one PAGE 25

1 $15x^2$	**2** $3x^2 + 1$	**3** $\dfrac{11}{(2x+5)^2}$	**4** $12x^2(x^3 + 1)^3$	**5** e^x
6 $2e^x$	**7** $10e^x$	**8** $e^x + 6x + 3x^2$	**9** $5e^{5x}$	**10** $12e^{4x}$
11 $6e^{2x}$	**12** $6e^{3x} + 6e^{2x}$	**13** $3^4 = 81$	**14** $6^3 = 216$	**15** $2^{-2} = 0.25$
16 $a^c = b$	**17** $a^b = c$	**18** $b^c = a$	**19** $c^b = a$	**20** $x^5 = 2$
21 $\log_2 8 = 3$	**22** $\log_5 25 = 2$	**23** $\log_4 0.25 = -1$	**24** $\log_2 0.125 = -3$	**25** $\log_7 y = x$
26 $\log_a p = 2$	**27** $\log_{10} z = y$	**28** $\log_e x = y$	**29** 5	**30** 3
31 1	**32** 3	**33** 6	**34** 2	**35** 3

36 2 **37** 0 **38** 1 **39** 3 **40** 0.5

41 $\ln 12 - 1$ **42** $\ln 25 - 2$ **43** $\ln 150 + 1$ **44** $\dfrac{\ln 34 - 1}{2}$ **45** $\ln 25 - 1$

46 $\ln 5, \ln 7$ **47** $\log(x^3 y)$ **48** $\log \dfrac{x^2}{y^3}$ **49** $\log \dfrac{a^2 b}{c^3}$ **50** $\log(1000x)$

51 $\ln(e^2 x)$ **52** $\ln \dfrac{e^3 y^2}{x}$ **53** 2026

54 **a** $(e - 1)$ m/s, (Approximately 1.72 m/s.) **b** $10(e - 2)$ m, (Approximately 7.18 m.)
c $10e^{0.1T}(e^{0.1} - 1) - 1$ m **d** 0.285 m **e** 1.587 m

Exercise 2A PAGE 32

1 $\dfrac{1}{x}$ **2** $\dfrac{1}{x}$ **3** $10x + \dfrac{1}{x}$ **4** $1 + e^x + \dfrac{1}{x}$ **5** $\dfrac{3}{3x+2}$

6 $\dfrac{2}{2x+3}$ **7** $\dfrac{2}{2x-3}$ **8** $\dfrac{2x}{x^2+1}$ **9** $-\tan x$ **10** $\dfrac{2}{x}$

11 $\dfrac{1}{3x}$ **12** $\dfrac{1}{2x}$ **13** $\dfrac{1}{x}$ **14** $\dfrac{2x+3}{x(x+3)}$ **15** $\dfrac{2x+1}{(x+4)(x-3)}$

16 $1 + \log_e x$ **17** $\dfrac{3}{x}(\log_e x)^2$ **18** $-\dfrac{1}{x}$ **19** $-\dfrac{1}{x(\log_e x)^2}$ **20** $e^x \log_e x + \dfrac{e^x}{x}$

21 $\dfrac{1 - \log_e x}{x^2}$ **22** $\dfrac{3(1 + \log_e x)^2}{x}$ **23** $\dfrac{1}{x} + \dfrac{1}{x+5} + \dfrac{1}{x+3} \left(= \dfrac{3x^2 + 16x + 15}{x(x+5)(x+3)} \right)$

24 $\dfrac{1}{x+1} - \dfrac{1}{x+3} \left(= \dfrac{2}{(x+1)(x+3)} \right)$ **25** $\dfrac{8x}{x^2+5}$ **26** $\dfrac{1}{x} - \dfrac{2x}{x^2-1} \left(= \dfrac{1+x^2}{x(1-x^2)} \right)$

27 $\dfrac{3}{x+2} - \dfrac{1}{x-2} \left(= \dfrac{2(x-4)}{(x+2)(x-2)} \right)$ **28** 7 **29** 3 **30** 7

31 -2 **32** $(4, \log_e 4)$ **33** $(0.5, -\log_e 4)$ **34** $(5, \log_e 25)$ **35** $(3, \log_e 18)$

36 $y = x - 1$ **37** $ey = x$ **38** $\dfrac{1}{x \ln 4}$ **39** $\dfrac{1}{x \ln 6}$

40 Approximate change in y is 0.5
$50 \ln 10.1 - 50 \ln 10 = 0.4975$, correct to four decimal places.

41 1.5 m/s, -0.25 m/s^2

42 Minimum point at $(5, 25 - 50 \ln 10)$

Exercise 2B PAGE 37

Note: At the time of writing the syllabus for this unit includes

$$\int \frac{1}{x}\,dx \text{ for } x > 0, \text{ and } \int \frac{f'(x)}{f(x)}\,dx \text{ for } f(x) > 0.$$

Thus whilst $\int \frac{1}{x}\,dx = \ln|x| + c,\ x \neq 0$, see the note on page 34, the restriction $x > 0$ (and $f(x) > 0$) makes the absolute value unnecessary.
Hence answers to this exercise are given here without the absolute value symbol.

1 $5 \ln x + c$ **2** $4 \ln x + c$ **3** $\dfrac{x^2}{2} + 2 \ln x + c$ **4** $\dfrac{1}{2} \ln x + c$

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5 $\ln(x^2+1)+c$ **6** $\frac{x^3}{3}+5\ln x+c$ **7** $2x^2+e^x+2\ln x+c$ **8** $2\ln(x+1)+c$

9 $4\ln(x^2-3)+c$ **10** $\ln(5x-3)+c$ **11** $5\ln(2x+1)+c$ **12** $3\ln(x^2+1)+c$

13 $\ln(x^2+x+3)+c$ **14** $3\ln(x^2+5x)+c$ **15** $10\ln(x^2+4)+c$ **16** $-\ln(\cos x)+c$

17 $\ln(\sin x)+c$ **18** $-\frac{1}{2}\ln(\cos 2x)+c$ **19** $-\ln(\cos x)+c$ **20** $-\frac{1}{5}\ln(\cos 5x)+c$

21 $-3\ln(\cos 2x)+c$ **22** $-\ln(\sin x+\cos x)+c$ **23** $\frac{1}{2}\ln(4x+\sin 2x)+c$ **24** $\ln(e^x+x)+c$

25 $\ln 3$ **26** $3\ln 1.5$ **27** $e^2-e+\ln 2$ **28** $x=\ln\left(\frac{t+2}{2}\right)$

29 $(4+\ln 3)$ units^2 **30** $(1-\ln 2)$ units^2 **31** $(2e\ln 2+1)$ units^2 **32** $\ln\left(\frac{2}{\sqrt{3}}\right)$ units^2

33 $a=3,\ b=5,\ 3\ln(x+4)+5\ln(x+2)+c$

34 **a** $e^{0.5}$ **b** $e^{0.25}$ **c** $2\ln\left(\frac{1+e^{0.5}}{2}\right)$

Miscellaneous exercise two PAGE 39

1 $\frac{dy}{dx}=2\cos 2x$ **2** $\frac{dy}{dx}=-3\sin 3x$ **3** $\frac{dy}{dx}=4e^{4x}$ **4** $\frac{dy}{dx}=20e^{4x}$

5 $\frac{dy}{dx}=\frac{5}{(x+1)^2}$ **6** $\frac{dy}{dx}=12(3x-1)^3$ **7** $\frac{dy}{dx}=\frac{2}{x}$ **8** $\frac{dy}{dx}=2x\log_e x+x$

9 $\frac{dy}{dx}=-\frac{1}{x^2}+6e^{2x}$ **10** $\frac{dy}{dx}=\frac{1+2x}{1+x+x^2}$ **11** $\frac{\log 11}{\log 2}$

12 **a** $2p$ **b** $3p+q$ **c** $p+2q$ **d** $p+0.5q$ **e** $p+q+3$ **f** $\frac{q}{p}$

13 **a** 4 **b** 8 **c** 2 **d** 2 **e** 8.5 **f** 34
g 0.5 **h** 8

14 **a** $p=xy$ **b** $p=x^y$ **c** $p=\frac{x^3}{y}$ **d** $p=100\sqrt{y}$

15 $e^2y=x+e^2$ **16** 0.88^tQ_0. The pump must work for approximately 23.4 minutes.

17 **a** $6x\ln(3x+2)+\frac{9x^2}{3x+2}$ **b** $1.8+6\ln 5$

18 **a** $A(e^{-1},0)$, $B(e,0)$ **b** $(1,-1)$ **c** Point $B(e,0)$ is the only point of inflection.

Exercise 3A PAGE 47

1 **a** $\frac{163}{186}$ **b** $\frac{64}{93}$ **c** $\frac{2}{93}$

2 **a** 36% **b** 15% **c** 24% **d** 48% **e** 24% **f** 53%

3 **a** 20 **b** **i** 0.82 **ii** 0.18 **iii** $\frac{8}{41}$

4 **a** 0.65 **b** 0.65 **c** 0.825 **d** $\frac{5}{26}$ **e** 0.8

5 **a** 0.28 **b** 0.72 **c** 0.67 **d** 0.64

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Exercise 3B PAGE 54

1 0.25 **2** 0.05 **3** 2 **4** 2.5 **5** 1.8 **6** 4.5

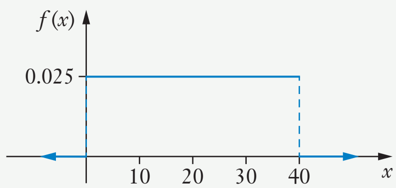
7 15 **8** 17.5 **9** $f(x) = \begin{cases} 0.5 & \text{for } 1 \le x \le 3 \\ 0 & \text{for all other values of } x. \end{cases}$

10 **a** 0.25 **b** 0 **c** 0.75 **d** $\frac{2}{3}$

11 **a** 1 **b** 0.3 **c** 0 **d** 1 **e** 0.625

12 **a** 25 **b** 0 **c** 0.4 **d** 0.4 **e** 0.8 **f** 1
g 0.2 **h** 0

13



a 0.5 **b** 0.625 **c** 0.2

Exercise 3C PAGE 59

1 0.5 **2** 0.25 **3** 1.25 **4** 2.5 **5** $\frac{1}{15}$ **6** $\sqrt{\frac{2}{\pi}}$

7 0.5 **8** 1.8 **9** 1.6 **10** 2.5 **11** 24 **12** $\frac{4}{15}$

13 $\frac{1}{\sqrt{e}-1}$ **14** 2.4 **15** 0.25 **16** 2

17 **a** $f(x) = \begin{cases} 0.125x & \text{for } 0 \le x \le 4 \\ 0 & \text{for all other values of } x. \end{cases}$ or $f(x) = \begin{cases} 0.125x & \text{for } 0 < x \le 4 \\ 0 & \text{for all other values of } x. \end{cases}$

b $f(x) = \begin{cases} 0.25 & \text{for } 1 \le x \le 2 \\ 0.5 & \text{for } 2 < x < 3 \\ 0.25 & \text{for } 3 \le x \le 4 \\ 0 & \text{for all other values of } x. \end{cases}$

18 **a** 0.25 **b** 0.4375 **c** 1 **d** $\frac{5}{9}$

19 **a** 0.5 **b** 0.125 **c** 0.875 **d** $\frac{3}{7}$

20 $h(x)$ could be a probability density function for $0 \le x \le 5$.

$\int_0^5 f(x)\,dx \ne 1$. Thus $f(x)$ cannot be a probability density function for $0 \le x \le 5$.

$g(x)$ is negative for $\frac{25}{6} < x \le 5$.

Thus $g(x)$ cannot be a probability density function for the interval $0 \le x \le 5$.

21 **a** 0.84 **b** 0.28 **c** $\frac{7}{16}$

22 **a** 0.08 **b** 0.64 **c** 0.48 **d** 0.75

23 **b** 0.25 **c** 0.15 **d** 0.6

24 **a** $-\frac{8}{3}$ **b** No. For some values in $1 \le x \le 4$, $f(x)$ is –ve. Thus $f(x)$ cannot be a pdf for $1 \le x \le 4$.

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25 **a** 0.75 **b** 0.5 **c** 0.84 **d** 1.87

26 $a = -5, b = 6$

27 **a** 0.4 **b** $\frac{2}{3}$

28 **a** $\frac{2}{9}$ **b** $\frac{5}{9}$ **c** $\frac{8}{9}$

29 **a** 0.8 **b** 0.8 **c** 0.95

30 **a** 0.2019 **b** Approximately 0.25 $(= {}^6C_2\ (0.2019)^2\ (0.7981)^4)$

Exercise 3D PAGE 70

1 Mean 4.5, variance $\frac{25}{12}$. **2** Mean 0.75, variance $\frac{3}{80}$. **3** Mean 0.25, variance $\frac{3}{80}$. **4** Mean 2.4, variance $\frac{192}{175}$.

5 **a** $\frac{16}{3}$ **b** $\frac{5\sqrt{2}}{3}$

6 100 metres. **7** Mean 3, variance 0.8, standard deviation $\sqrt{0.8}$.

8 Mean 6.5 (i.e. 4.5 + 2), variance $\frac{25}{12}$ (i.e. no change).

9 Mean 9 (i.e. 4.5×2), variance $\frac{25}{3}$ (i.e. $\frac{25}{12} \times 2^2$).

10 **a** $E(Y) = 36, SD(Y) = 9$ **b** $E(Y) = 15, SD(Y) = 3$ **c** $E(Y) = 29, SD(Y) = 6$

11 **a** $E(Z) = 102, SD(Z) = 20$ **b** $E(Z) = 45, SD(Z) = 8$ **c** $E(Z) = 64, SD(Z) = 12$

12 Mean 118.4, variance 51.84, standard deviation 7.2

13 Mean ÷ 100, standard deviation ÷ 100.

In questions **14** to **19** the choice of ≤ or <, and ≥ or >, could differ from that shown here.

14 $P(X \le x) = \begin{cases} 0 & \text{for } x \le 0 \\ 0.25x & \text{for } 0 < x \le 4 \\ 1 & \text{for } x > 4 \end{cases}$

15 $P(X \le x) = \begin{cases} 0 & \text{for } x \le 2 \\ 0.25(x-2) & \text{for } 2 < x \le 6 \\ 1 & \text{for } x > 6 \end{cases}$

16 $P(X \le x) = \begin{cases} 0 & \text{for } x \le 0 \\ x^3 & \text{for } 0 < x \le 1 \\ 1 & \text{for } x > 1 \end{cases}$

17 $P(X \le x) = \begin{cases} 0 & \text{for } x \le 1 \\ \ln x & \text{for } 1 < x \le e \\ 1 & \text{for } x > e \end{cases}$

18 $P(X \le x) = \begin{cases} 0 & \text{for } x \le 0 \\ 1 - e^{-x} & \text{for } x > 0 \end{cases}$

19 $P(X \le x) = \begin{cases} 0 & \text{for } x < 5 \\ 0.5x - 0.02x^2 - 2 & \text{for } 5 < x \le 10 \\ 1 & \text{for } x \ge 10 \end{cases}$

20 **a** 0.7 **b** 0.3 **c** 0.4 **d** 0.7

21 **a** 0.7364 **b** 0.2636 **c** 0.2835 **d** 0.7165 **e** 0.1184

Miscellaneous exercise three PAGE 73

1 $\frac{\log 6}{\log 3}$

2 **a** 0.6 **b** 0.2 **c** 0.5

3 **a** $q - p$ **b** $2p + q$ **c** $p + 2q$ **d** $3p + 1$ **e** $\frac{q}{p}$ **f** $\frac{p}{q}$

4 **a** $\ln\left(\frac{17}{e+1}\right)$ **b** $\frac{7\ln 50 + 1}{\ln 50 - 2}$

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5 **a** ${}^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 \approx 0.032\,15$ **b** $\left(\frac{1}{6}\right)^3 \approx 0.004\,63$ **c** 0.003 34 (5 dp) **d** 0.180 87 (5 dp)

6 $\frac{1}{x}$ **7** $3+\frac{1}{x}$ **8** $\frac{2}{x}$ **9** $\frac{6}{x}$ **10** $\frac{1}{2x}$ **11** $-\frac{1}{x}$

12 Approximately 31 years.

13 **a** $(2, 2 + \ln 4)$ **b** $(3, \ln 18)$

14 **a** $\frac{200}{1+x}$ dollars per unit **b** \$15.36

15 $y = \sqrt{3}x - \frac{\sqrt{3}\pi}{6}$

16 **a** 0.6321 **b** 0.9502 **c** 0.0498

17 $(0.5, 0.5 + \log_e 2)$, minimum. **18** Mean 4, variance 16

Exercise 4A PAGE 78

1 **a** 1 **b** 1.7 **c** –2 **d** 0.5 **e** –0.75

2 Test A: 2.5, Test B: –1, Test C: 1.25, Test D: 0.2

3 Computing (1.216), Chemistry (0.278), Mathematics (–0.385), Electronics (–0.616)

4 English, Mathematics, Science, Social Studies.

5 Jill: *'Well I got 1'*

Jill: *'The mean was zero.'*

Jill: *'Oh he got –0.25 .'*

Exercise 4C PAGE 90

1 0.6915 **2** 0.9088 **3** 0.8849 **4** 0.5793 **5** 1.73 **6** 37.64

7 7.54 **8** 21.25 **9** 0.2266 **10** 0.6377 **11** 54.56

12 **a** 0.5828 **b** –0.6433 **c** 1.2265 **d** –0.7388

13 **a** 19.5 **b** 21.9 **c** 18.7 **d** 23.1

14 **a** 0.68 **b** 0.95 **c** 0.997 **d** 0.95 **e** 0.997 **f** 0.34

g 0.84 **h** 0.16 **i** 0.84 **j** 0.16

15 **a** 99.7% **b** 16% **c** 13.5%

16 **a** 16% **b** 2.5%

17 **a** 0.3085 **b** 0.0062

18 **a** 0.5 **b** 0.34

19 **a** 0.3085 **b** 0.2902 **c** 0.0228

20 **a** approximately 11 **b** approximately 11 **c** approximately 39

21 0.0548

22 **a** 415 **b** 217 **c** 55.5 (nearest half mark) and 68.5 (nearest half mark).

23 To nearest 0.5 cm: 158.5 cm, 191.5 cm

24 A/B: 78, B/C: 68, C/D: 55, D/F: 47

25 **a** 0.842 standard deviations **b** 44.2

26 **a** 0.1587 **b** 7:38 a.m. **c** 7:33 a.m.

27 **a** 2 years **b** 7 years **c** 91 years **d** 0.783

28 **a** approximately 40 **b** 0.236

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29 **a** 0.036 **b** 0.242 **c** 0.31
30 **a** 0.2054 **b** 0.0066
31 **a** 505 **b** 255
32 **a** approximately 2.3% **b** 4.6

Exercise 4D PAGE 96

1 For normally distributed data we would expect approximately 68% of the data points to be within one standard deviation of the mean. The fact that 80% meet this criteria would indicate that it could be unwise to model the data set as being normally distributed.

2 For $X \sim N(4.37, 2.52^2)$, $P(X > 8) = 0.0749$, i.e. approximately 7.5% of the measurements are above 8. The data set mentioned in this question has more than 14% of the measurements greater than 8 thus indicating that to assume the data set to be normally distributed could be unwise.

3 For $X \sim N(8.9, 3.1^2)$, $P(5.8 < X < 12.0) = 0.6827$, i.e. approximately 68% of the measurements are within one standard deviation of the mean. We have just 31% within one standard deviation thus indicating that to assume the data set to be normally distributed could be unwise.

4 The given information shows that the data distribution is not symmetrical. For a normal distribution we would expect about 34% of measurements between the mean and one standard deviation below the mean and 34% between the mean and one standard deviation above the mean. We have 26% and 44% respectively. Thus to assume the data set to be normally distributed could be unwise.

5 The given information shows that the measurements are not symmetrically distributed.

Also, for $X \sim N(\bar{x}, \sigma^2)$, $P(X > \bar{x} + 2\sigma) = 0.022\,75$, i.e. approximately 4 measurements would be more than two standard deviations above the mean and approximately 4 would be more than two standard deviations below the mean. We have ten above and none below. Thus to assume the data set to be normally distributed could be unwise.

6 **a** Yes

b Depends what 'sufficiently' reliable means. Without knowing what the probabilities are to be used for we cannot judge if they are likely to be 'sufficiently' reliable. With 520 measurements and the scientist's 3% rule the approximation to normal is likely to be good but her rule does not allow for any unusual grouping within each interval that might make the approximation questionable. (Having the 520 measurements she could carry out further checks for this aspect.)

7 If the data is normally distributed, from the symmetry of such a distribution, we should have the mean close to the median, which is the case for this data, and the lower quartile should be approximately as far below the mean as the upper quartile is above it.

Again this is the case for this data, $69.2 - 44.7 \; (= 24.5) \approx 44.7 - 19.3 \; (= 25.4)$.

For $X \sim N(0, 1^2)$ solving $P(X \le x) = 0.25$ gives $x = -0.67$

Hence, in a normal distribution the lower quartile is 0.67 standard deviations below the mean. For our data the lower quartile is 0.96 standard deviations below the mean.

Similarly, in a normal distribution the upper quartile is 0.67 standard deviations above the mean. For our data the upper quartile is 0.93 standard deviations above the mean.

The figures suggest that the data is not normally distributed.

8 **a** By binomial 0.8697
By normal 0.8692

b By binomial 0.0563
By normal 0.0564

c By binomial 0.0944
By normal, i.e. $P(20.5 < X < 25.5)$ with $X \sim N(30, 12)$, 0.0939

Miscellaneous exercise four PAGE 98

1 $\frac{1}{x}$ **2** $\frac{10}{x}$ **3** $\frac{-1 + \ln x}{(\ln x)^2}$ **4** $\frac{6x}{x^2 + 1}$ **5** $\frac{2(x + 2)}{(x + 1)(x - 1)}$ **6** $\frac{1}{x \ln 5}$

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7 0.0668 **8** 0.6827 **9** 1.32 **10** 18.25 **11** $\frac{3}{e}$ **12** 2

13 0.0038

14 **a** −0.202 **b** −1.126 **c** 0.332 **d** −0.228

15 **a** Approximately 17 days **b** Approximately 237 days **c** Approximately 197 days.

16 $a = 0.3$, $k = 0.1$, $E(X) = 1.4$, $VAR(X) = \frac{17}{75}$.

17 **a** 0.5 **b** $\frac{\sqrt{2}}{2}$

18 **a** $P(x) \approx 1000x - 25\,000 + 20\,000\log_e\left(1 - \frac{x}{100}\right)$, $x < 100$.

b Extract 80 kg per 5 tonne batch for a maximum profit of approximately \$22 800.

19 **a** $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{11}{18}$ **d** 0

20 **a** 0.0304 **b** 0.116

Exercise 5A PAGE 109

1 **a** Likely to introduce bias. **b** Likely to introduce bias. **c** Not likely to introduce bias.

d Not likely to introduce bias. (Not that is likely to influence car colour anyway.)

e Not likely to introduce bias. (Not that is likely to influence height anyway, unless perhaps the school has something like a special basketball intake.)

2 Two under twenty, three in their twenties, four in their thirties and one of 40 or over.

4 19 year 8s, 19 year 9s, 18 year 10s, 12 year 11s and 12 year 12s.

5 Approximately 190. **6** Approximately 760. **7** Approximately 3200.

8 Either by considering the symmetry of the situation, or from the calculation shown below, the long term mean, or expected value, for rolling a fair normal die is 3.5.

$$1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = 3.5$$

With one mean, 4.08 (= 3.5 + 0.58), noticeably further from 3.5 than the other (3.42 = 3.5 − 0.08) we would expect the one closer to the expected value to be the one involving the 150 rolls. This suggests that Christine, with her 150 rolls of the die, would have the mean of 3.42 and Shane, with his 12 rolls of the die, would have the mean of 4.08.

9 Either by considering the symmetry of the situation, or from the calculation shown below, the long term mean, or expected value, for the sum of the two numbers obtained by rolling two normal fair dice is 7.

$$2 \times \frac{1}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} + 5 \times \frac{4}{36} + 6 \times \frac{5}{36} + 7 \times \frac{6}{36} + 8 \times \frac{5}{36} + 9 \times \frac{4}{36} + 10 \times \frac{3}{36} + 11 \times \frac{2}{36} + 12 \times \frac{1}{36} = 7.$$

With one mean, 6.42 (= 7 − 0.58) noticeably further from 7 than the other (7.17 = 7 + 0.17) we would expect the one closer to the expected value to be the one involving the 150 rolls. This suggests that Horace, with his 150 rolls of the two dice, would have the mean of 7.17 and Portia, with her 12 rolls of the two dice, would have the mean of 6.42

10 Approximately 6500 people did not complete a census form.

11 Ask someone to read and comment about your explanation and you read and comment about their explanation.

Have they explained why the process works or have they just listed the steps involved?

Have they considered possible sources of errors, for example:

Will the tagged turtles remix with the rest of the population?

Will 'once caught turtles' be more prone to recapture?

Will the initial capture involve a random selection from the lake? What if just one region of the lake is used for capture and recapture? etc.

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Counting seals PAGE 111

Explain how we could 'randomly select' the squares to be photographed and suggest how many squares should be selected.

Number the squares and use a random process to select at least 30, let us say 50, squares from the ones entirely covering land, as per the plan. This could be done using a random number generator set to generate integers from 1 to 900 and record the numbers, ignoring any repeated numbers and any that represented squares that were not 'all land' until 50 different 'all land' squares were chosen.

Will your random selection guarantee that the sample is an accurate representation of the population of seals on the island at the time the photographs were taken? Explain.

We cannot guarantee that the sample is an accurate representation of the population but if the land only squares are representative of the whole island then the number of squares in our sample allows us to be reasonably confident that the results from the sample can be used to give a reasonable population estimate.

How could the 'seal counts' from the selected photographs be used to estimate the seal population on the island at the time the photographs were taken?

Using our random squares to determine an average number of seals per 10 000 m^2 we multiply this by 734 to estimate the number on the 7 340 000 m^2 (= 7.34 km^2) island.

Suggest any improvements that could be made to the plan?

The squares that contain some water and some land will be near the ocean and, given the somewhat laboured nature of a seal's movement on land, many seals may choose to stay near the water's edge. Hence these squares, with their proximity to the water, could well be where many seals choose to rest, choosing not to struggle further inland. Thus whilst it is perhaps wise to treat these 'part water part land' squares differently, dismissing them from the sample altogether may not be the best option. Better to sample these water's edge squares too and then include the data from them in some proportional way.

Miscellaneous exercise five PAGE 123

1 5

2 $\log_e\left(\frac{P}{9}\right) - 1$ **a** 1.996 **b** 4.991 **c** 2

3 **a** $p + q$ **b** $p - q$ **c** $2p + 3q$ **d** $0.5p$ **e** $\frac{p}{\log e}$ **f** $\frac{2q}{1 - \log 2}$

4 $n = 150$, $p = 0.4$, $P(X \le 50) = 0.056$ (correct to 3 dp).

5 $1 + \ln(5x)$ **6** $\frac{2\log_e x}{x}$ **7** $x + 2x\ln x$ **8** $\frac{2(3 + \ln x)}{x}$ **9** $\frac{2(x-1)}{x^2}$ **10** $-\frac{1}{x(\ln x)^2}$

11 $f''(x) = 5x + 6x\ln x$

12 **a** 1.25 **b** 2

13 **a** $k = 0.25$ **b** $\frac{11}{6}$ **c** $\frac{11}{36}$ **d** $\frac{\sqrt{11}}{6}$

e $$P(X \le x) = \begin{cases} 0 & \text{for} \quad x < 1 \\ -\frac{1}{8}x^2 + x - \frac{7}{8} & \text{for} \quad 1 \le x \le 3 \\ 1 & \text{for} \quad x > 3. \end{cases}$$

(Placement of the 'equals part of the inequality' could vary from that shown here.)

Or, without actually performing the integration, this could be written:

$$P(X \le x) = \begin{cases} 0 & \text{for} \quad x < 1 \\ \int_1^x 0.25(4 - t)\,dt & \text{for} \quad 1 \le x \le 3 \\ 1 & \text{for} \quad x > 3. \end{cases}$$

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14 First recapture suggests 3768 birds of this species in the area.

Second recapture suggests 3143 birds of this species in the area.

Estimated number of birds of this species in the area: Approximately 3500.

A possible problem with using capture-recapture techniques on migratory birds would occur if the swampy area was just a short stay location during the migration. If this was the case some of the birds tagged in the first batch may have moved on by the time of the recapture. The proportion of tagged ones in the second, and subsequent captures may not then reflect the proportion tagged in the whole population. However if it were known that almost all of the birds stayed in the swampy area for a reasonable amount of time, and that the capturing and recapturing could all be carried out during this time, this problem would be avoided.

15 Only the last four parts, **i**, **j**, **k** and **l**, are true for all $p > 0$ and $q > 0$.

16 0.6205

17 **a** 45.9 **b** 59.2 **c** 48.2 **d** 43.3

18 **a** 28 is 0.385 standard deviations from the mean (below).

b The mean is 30.21 (2 dp).

Exercise 6A PAGE 142

1 First survey has $\hat{p} = \frac{49}{123} \approx 0.398$ Second survey has $\hat{p} = \frac{761}{2348} \approx 0.324$

The second value for $\hat{p}$ is likely to be the better estimate of p, the population proportion, due to it involving a larger sample.

2 Assuming the samples are reasonably representative of the shoppers using this supermarket (which might not be the case given all of the samples were on just one day), and with each sample involving the same number of people, we can estimate the population proportion by finding the mean of the sample proportions. This gives an estimate of 0.72.

3 **a** By adding the numbers of samples $(1 + 4 + 6 + 7 + 2 + 7 + 2 + 2 + 4)$ we obtain the total number of samples as 35.

b By finding the mean of the sample proportions we obtain an estimate of the population proportion of 0.799.

4 **a** 0.225 **b** 0.25 **c** Mean of $\hat{p}$ is 0.25, standard deviation is $\sqrt{\frac{0.25(1-0.25)}{320}} \approx 0.024$.

5 **a** $\frac{13}{18}$ **b** 0.67 **c** Mean of $\hat{p}$ is $\frac{13}{18}$, standard deviation is $\sqrt{\frac{\frac{13}{18}(1-\frac{13}{18})}{100}} \approx 0.0448$

d Our value for $\hat{p}$ is approximately 1.166 standard deviations below the mean value, p.

6 **a** The population proportion is 0.84 (or 84%).

b The sample proportion is 0.6125 (or 61.25%).

c With $p = 0.84$ and $n = 240$ we would expect $\hat{p}$ to be normally distributed with mean 0.84 and standard deviation 0.024. A $\hat{p}$ value of 0.6125 is more than 9 standard deviations below the mean! Such an extraordinary result suggests that the sample was not truly representative of the population as a whole with regard to household internet access.

7 **a** For $p = 0.1$ and a sample size of 1000 we would expect the sample proportions of left handers to be normally distributed with mean 0.1 and standard deviation 0.0095 $(= \sqrt{\frac{0.1 \times 0.9}{1000}})$. Our sample proportion of 0.112 is just 1.26 standard deviations above the mean. Thus, whilst it is not clear whether the classification of the school students as being 'left handed' was the same as the 'ranging from moderate through strongly left handed' classification the paper mentions, the proportion of left handers in the sample seems consistent with what we might expect for a sample size of 1000 and population proportion 0.1.

b Not knowing the number of participants in the 1981 World Championship foil competition means that we do not know the sample size. However, even not knowing the sample size, the proportion of 0.35 is so much higher than the population proportion of 0.1. The participants in the 1981 World Championship foil competition do not form a representative sample of the left handedness in the general population.

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8 **a** The sample proportion is $\frac{461}{1247} \approx 0.37$

b An estimate of the standard deviation of the sample proportions is $\sqrt{\frac{\frac{461}{1247}(1 - \frac{461}{1247})}{1247}} \approx 0.0137.$

9 **a** The sample proportion is $\frac{143}{248} \approx 0.577.$

b An estimate of the standard deviation of the sample proportions is $\sqrt{\frac{\frac{143}{248}(1 - \frac{143}{248})}{248}} \approx 0.0314.$

10 **a** Simply finding the mean of the sample proportions takes no account of the fact that the proportions for larger samples should give a better estimate than those with smaller samples. We need to attach more importance to the sample proportions coming from the larger samples.

b If we use the given information to determine the number in each sample with high blood pressure, then express the total number with high blood pressure as a proportion of the total number surveyed, this would give a better estimate of the population proportion.

A better estimate for the population proportion would be $\frac{193}{756} \approx 0.2553.$

(Note: Simply averaging the sample proportions gives 0.3275.)

Sample	Number in sample	Sample proportion having high blood pressure	Number in sample having high blood pressure
1	8	0.5	4
2	10	0.1	1
3	50	0.28	14
4	25	0.24	6
5	10	0.2	2
6	80	0.2375	19
7	56	0.286	16
8	10	0.1	1
9	50	0.2	10
10	180	0.261	47
11	20	0.35	7
12	10	0.1	1
13	8	0.375	3
14	25	0.2	5
15	8	0.5	4
16	150	0.2	30
17	20	0.3	6
18	25	0.32	8
19	10	0.8	8
20	1	1	1
Total	756		193

11 For $n = 200$ and $p = 0.\ 1$ we would expect the sample proportions to be well modelled by a normal distribution with mean 0.1 and standard deviation $\sqrt{\frac{0.1(1 - 0.1)}{200}} \approx 0.021\,21.$

With 35 seeds (or more) from a sample of 200 unable to germinate the sample proportion is 0.175 (or more). This is more than 3.5 standard deviations above the mean of the distribution, $0.175 \approx 0.1 + 3.54 \times 0.02\ 12$, and is therefore extremely unlikely.

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12 We expect the distribution of sample proportions to approximate to a normal distribution if $np \geq 10$ and $n(1-p) \geq 10$.

This is the case with Graph One with $np = 50 \times 0.5 = 25$ and $n(1-p) = 50 \times 0.5 = 25$

However, with Graph Two, $np = 50 \times 0.05 = 2.5$

Hence it should not be a surprise that Graph One better approximates the shape of the normal distribution.

13 0.45 and 0.55.

14 Even if the politician's claim is correct, in a sample of 200 people we would not expect to necessarily find that the proportion of people voting for the politician would exactly match the 52% claimed but we would expect the sample percentage to be quite close. Indeed we would expect the sample proportion to come from a normally distributed random variable of mean 0.52 and standard deviation 0.035. For such a distribution the sample proportion of 81 out of 200 (= 0.405) is approximately 3.3 standard deviations below the mean. This would be very unusual. Hence, if the sample of 200 people fairly represented the voting behaviour of the people who intended to vote in the election for the seat of Dasha we would have to question the politician's claim that he would get 52% of the vote.

15 There is a 90% chance that in a sample of 800 cars produced by this company the sample proportion that are blue will be between 21.5% and 26.5%.

Exercise 6B PAGE 154

Note
- The accuracy stated here is to allow you to check your answers. In practice values could well be rounded more heavily, dependent upon the situation and what the statistics are to be used for. For example the 90% confidence interval in question 2 may well be quoted as 0.83 to 0.87, i.e. 0.85 ± 0.02, and the interpretation could well refer to 83% and 87%.
- Confidence intervals given here have been rounded using the usual regime for rounding, or according to the level of rounding stipulated in the question. However, if in a real situation it were crucial that when stating a 95% confidence interval we were not claiming to be greater than 95% confident, or perhaps not claiming to be less than 95% confident, the rounding regime would have to be more carefully applied.

1 The 95% confidence interval is 0.3476 to 0.4024. (I.e. 0.375 ± 0.0274).

Were we to repeat such sampling we could expect 95% of the 95% confidence intervals so formed to contain the population proportion. Hence, with 95% confidence we estimate that between 34.76% and 40.24% of the people living in Australia are in favour of the idea of introducing compulsory national service.

2 The 90% confidence interval is 0.8292 to 0.8708. (I.e. 0.85 ± 0.0208).

Were we to repeat such sampling we could expect 90% of the 90% confidence intervals so formed to contain the population proportion. Hence, with 90% confidence we estimate that between 82.92% and 87.08% of the people living in Australia who had recently contacted their bank for online help were either satisfied or very satisfied with the service they received.

3 The 99% confidence interval is 0.6904 to 0.8296. (I.e. 0.76 ± 0.0696).

Were we to repeat such sampling we could expect 99% of the 99% confidence intervals so formed to contain the population proportion. Hence, with 99% confidence we estimate that between 69.04% and 82.96% of the people regularly playing the particular sport agreed that the recent rule changes were a good idea.

4 70% ± 3%.

5 43.2% to 46.8%.

Were we to repeat such sampling we could expect 90% of the 90% confidence intervals so formed to contain the population proportion. Hence, with 90% confidence we estimate that between 43.2% and 46.8% of the people of the particular nation involved wanted to see changes to the current daylight saving rules.

For the particular community the sample proportion is 70% which is a long way outside of the 90% confidence interval. If the original sample was fairly representative of the population as a whole the 70% figure would suggest that the particular community returning the 70% proportion was not typical of the national opinion. Perhaps local considerations made this community much more inclined to want to see changes to the daylight saving rules.

6 Discuss and compare your statement with the statements made by others in your class.

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7 0.0218

8 The 99% confidence interval.

9 **a** The sample proportion of acceptable components is 0.82

b An example of the sort of statement that could be made:

With 90% confidence we estimate that around the time the sample was taken, between 77.5% and 86.5% (i.e. 82% ± 4.5%) of the components made by this machine were of an acceptable standard.

c An example of the sort of statement that could be made:

With 99% confidence we estimate that around the time the sample was taken between 75% and 89% (i.e. 82% ± 7%) of the components made by this machine were of an acceptable standard.

10 Rounding up to the next integer gives a sample size of 228.

11 Rounding up to the next integer gives a sample size of 752.

12 The sample size should be 350 or greater.

13 We can be 95% confident that of all Australian males between the ages of 20 and 30, between 72% and 80% are taller than their father.

To be 99% confident our interval would need to be larger.

To be more confident that p will lie in our interval the interval needs to be larger.

14 We can be 90% confident that the proportion of Australians having the particular attribute lies between 18% and 30%.

The sample size was 137 of whom 33 possessed the particular attribute.

15 **a** 0.35, or 35%.

b 0.0218

c 0.3073 to 0.3927, i.e. 0.35 ± 0.0427

Were we to repeat such sampling we could expect 95% of the 95% confidence intervals so formed to contain the population proportion. Hence we can be 95% confident that between 30.7% and 39.3% of year 12 Australian school students would say they intend to proceed to University the following year.

d 972 or greater.

Miscellaneous exercise six PAGE 156

1 **a** $x = 2$ **b** $x = 8$ **c** $x = 100$ **d** $x = 1$

2 **a** $\frac{3\sqrt{x}}{2}$ **b** $20x^4 + \frac{1}{x}$ **c** $\frac{7}{x}$ **d** $\frac{15x^2 - 6}{5x^3 - 6x}$

3 **a** $\frac{5}{5x - 1}$ **b** $\frac{4x^3}{x^4 + 1}$ **c** $\frac{2x}{x^2 - 1}$

4 $y = 3 - \frac{x}{e}$

5 **a** 0.25 **b** 0.2875 **c** Mean of $\hat{p}$ is 0.25, standard deviation is $\sqrt{\frac{0.25(1 - 0.25)}{160}} \approx 0.0342$.

d Our value for $\hat{p}$ is approximately 1.1 standard deviations above p.

6 **a** 0.125 **b** 0.75 **c** 0.75 **d** 0.8

7 0.9179 **8** $f''(x) = -\frac{5}{x^2}$, $f(x) = x + 5\ln x + 4$

9 **a** $\ln 5$ **b** $\frac{\ln 2}{\ln 5}$

10 30

11 **a** 0.003 **b** 0.006

12 **a** 0.76 **b** $\mu = 33.9$, $\sigma = 7.2$ **c** 0.95

13 Compare your answer with those of others in your class.

14 63.5%

15 Discuss your results with those of others in your class.

16 Were we to repeat such sampling we could expect 95% of the 95% confidence intervals so formed to contain the population proportion. Hence we can be 95% confident that between 17% and 21% of adult Australians were, at the time of the survey, 'mobile only'.

The proportion for the Australians aged 65 or over, 0.04 or 4%, and for Australians in their twenties, 0.42 or 42%, are both a long way outside the 17% to 21% interval that we would expect the population proportion to lie, and would therefore expect other sample proportions to be quite close to. The figures suggest that samples considering only Australians over 65 or only Australians in their twenties would not be representative of the population as a whole, as we would probably expect when considering this 'mobile only' category. Compared to the populations as a whole, those in their twenties are more likely to be 'mobile only' and those over 65 are less likely to be 'mobile only'. For this 'mobile only' attribute, the '65 and over' sample and the 'in their twenties' sample are not representative of the population as a whole.

17 With a population proportion of 0.22 and sample size of 400 we would expect the sample proportions to be normally distributed with mean 0.22 and standard deviation of approximately 0.0207. Thus 20%, or 0.2 is approximately one standard deviation from the mean. The change could be explained as being reasonable variation in sample proportions.

If the survey had involved 4000 people we would now expect the sample proportion to be from a normal distribution with mean 0.22 and standard deviation 0.006 55. The sample proportion of 20%, or 0.2 is then just over 3 standard deviations from the mean – very unlikely to occur just on the basis of random variations and now much more likely to indicate a loss of popularity.

18 For a sample size of 100 and sample proportion of faulty batteries of 0.15 we would expect the sample proportions to be approximately normally distributed with a mean equal to the population proportion and a standard deviation of $\sqrt{\frac{0.15 \times 0.85}{100}}$, i.e. approximately 0.036. If the population proportion is as claimed, i.e. 0.15, then 0.17 is only 0.56 of a standard deviation above this, certainly possible for the stated population proportion. Thus the fact that a sample of 100 had 17 faulty batteries does not mean the claim that the population proportion of faulty batteries is 0.15 is necessarily false.

However there is the issue that the advertising might suggest to the customer that when they buy 100, as Joe did, they will be getting 85 that do work. Perhaps it should be suggested that the company tests the batteries and only sells working ones, or maybe that the advertising wording be changed so that customers realise that they could end up with less than (or maybe more than) 85% that work.

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